Measurement of Lift and Drag in the Laminar Wind Tunnel

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The measurement of the aerodynamic coefficients of a two dimensional model in the LWK is fully automated. So the corrected polar is plotted on line and can be observed on a plotter and a screen. During integration of the drag along the span the variation of drag can be controlled.

The total installation is calibrated by exact pressures to the end of the results on the plotter and screen before measuring a polar.
The zeroes of the pressure transducers are controlled and corrected (if necessary) as well.

The test results compare well with those of other facilities /5/6/.

Measurement of lift:
The experimental setup for lift measurement

Measurement of drag
The experimental setup for drag measurement

Experimental integration

Measurement of lift.

The lift of the model is similar to the Langley Low Turbulence Tunnels /1 / obtained by integrating the pressure distributions along the tunnel walls opposite to the model surfaces. As the integration is performed only over a restricted length instead of infinity in upstream and downstream directions a correction factor η is applied that gives the ratio of the integrated lift to the actual lift / 2/. First the correction factor η_x applying to a point vortex was found then the weighted average of this factor over the model chord was estimated. This single vortex is assumed on the axis of the test section at x=0 , y=0. To meet the condition that only a tangential velocity u along the walls exists (v=0) an infinite vertical row of vortices of alternating sign is necessary see figure 1.

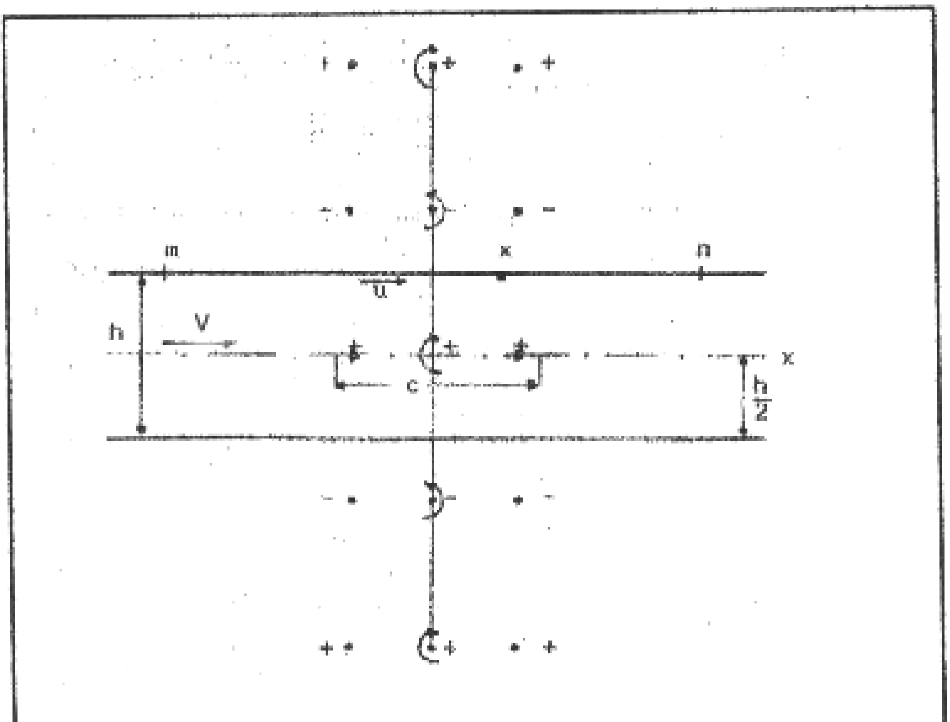

Figure 1

The complex potential function f of this image system can be thought to be composed of the potential function f_1 formed by all vortices with positive sign and distance 2h to the origin and the potential function f_2 of the vortices with negative sign with their origin shifted by $i \cdot h$ see figure 2.

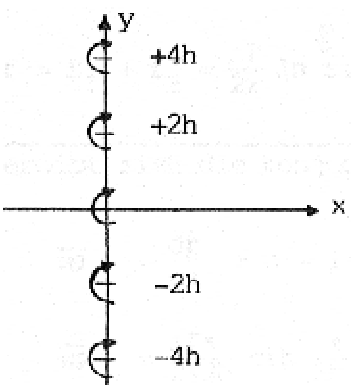

Figure 2

Γ strength of a single vortex
z complex variable (x + iy)
h tunnel height

The potential function f_1 of the positive vortices:

A finite number of vortices with strength $\frac{\Gamma}{2\pi}$ is located at y=0, ±2h, ±4h, … ±n 2h

The potential function of these vortices is

$$f_1 = \frac{i \cdot \Gamma}{2\pi} \cdot \ln z + \frac{i \cdot \Gamma}{2\pi \cdot} \cdot \ln(z - 2ih) + ... + \frac{i \cdot \Gamma}{2\pi} \cdot \ln(z - n \cdot 2ih)$$

$$+ \frac{i \cdot \Gamma}{2\pi \cdot} \cdot \ln(z + 2ih) + ... + \frac{i \cdot \Gamma}{2\pi} \cdot \ln(z + n \cdot 2ih) \qquad (1)$$

$$f_1 = \frac{i \cdot \Gamma}{2\pi} \cdot \ln\left[z \cdot \left(z^2 + (2h)^2\right) \cdot \left(z^2 + 2^2 \cdot (2h)^2\right) .... \cdot \left(z^2 + n^2 \cdot (2h)^2\right)\right]$$

$$f_1 = \frac{i \cdot \Gamma}{2\pi} \cdot \ln\left[\frac{\pi \cdot z}{2h} \cdot \left(1 + \frac{z^2}{(2h)^2}\right) \cdot \left(1 + \frac{z^2}{2^2 \cdot (2h)^2}\right) .... \cdot \left(1 + \frac{z^2}{n^2 \cdot (2h)^2}\right)\right]$$

$$+ \frac{i \cdot \Gamma}{2\pi} \cdot \ln\left[\frac{2h}{\pi} \cdot (2h)^2 \cdot 2^2 \cdot (2h)^2 ..... \cdot n^2 \cdot (2h)^2\right]$$

the last term which is constant can be dropped.

Now

$$\sinh x = x \cdot \left(1 + \frac{x^2}{\pi^2}\right) \cdot \left(1 + \frac{x^2}{2^2 \pi^2}\right) .... \cdot \left(1 + \frac{x^2}{n^2 \pi^2}\right)$$

for n $\to \infty$ and $\frac{\pi \cdot \cdot z}{2h} = x$ $\quad \frac{z^2}{(2h)^2} = \frac{x^2}{\pi^2}$

$$f_1 = \frac{i\Gamma}{2\pi} \cdot \ln \sinh \frac{\pi \cdot z}{2h} \qquad (2)$$

The potential function f_2 of the vortices with negative sign which is shifted by $i \cdot h$ along the y-axis is

$$f_2 = -\frac{i\Gamma}{2\pi} \cdot \ln \sinh \frac{\pi(z - i \cdot h)}{2h} \qquad (3)$$

and the potential function for the total vortex system is

$$f = f_1 + f_2 = \frac{i\Gamma}{2\pi} \cdot \ln \sinh \frac{\pi \cdot z}{2h} - \frac{i\Gamma}{2\pi} \ln \sinh \frac{\pi(z - i \cdot h)}{2h} \qquad (4)$$

The complex velocity is

$$\varpi = \frac{df}{dz} = u - i \cdot v$$

$$\varpi = \frac{i\Gamma\pi}{2\pi\cdot 2h}\cdot ctgh\frac{\pi}{2h}\cdot z - \frac{i\Gamma\pi}{2\pi\cdot 2h}\cdot ctgh\frac{\pi(z-i\cdot h)}{2h}$$

$$\varpi = \frac{i\Gamma}{4h}\cdot\left(ctgh\frac{\pi}{2h}\cdot z - ctgh\frac{\pi}{2h}(z-i\cdot h)\right) \qquad (5)$$

To calculate the pressure distribution along the tunnel walls the induced velocity at

$z = x + i\cdot\frac{h}{2}$ must be found.

$$\varpi = \frac{i\Gamma}{4h}\cdot\left(ctgh\frac{\pi}{2h}\cdot\left(x+i\frac{h}{2}\right) - ctgh\frac{\pi}{2h}\left(x+i\frac{h}{2}-i\cdot h\right)\right)$$

$$\varpi = \frac{i\Gamma}{4h}\cdot\left(ctgh\left(\frac{\pi}{2h}x+i\cdot\frac{\pi}{4}\right) - ctgh\left(\frac{\pi}{2h}x-i\cdot\frac{\pi}{4}\right)\right)$$

$$\varpi = \frac{i\Gamma}{4h}\cdot\left(ctgh\cdot i\cdot\left(-i\cdot\frac{\pi}{2h}x+\frac{\pi}{4}\right) - ctgh\cdot i\cdot\left(-i\cdot\frac{\pi}{2h}x-\frac{\pi}{4}\right)\right)$$

$$\varpi = \frac{i\Gamma}{4h}\cdot\left(ctgh\cdot i\cdot\left(\frac{\pi}{4}-i\cdot\frac{\pi}{2h}x\right) + ctgh\cdot i\cdot\left(\frac{\pi}{4}+i\cdot\frac{\pi}{2h}x\right)\right) \qquad (6)$$

with ctgh ix = – i ctg x

$$\varpi = \frac{i\Gamma}{4h}\cdot\left[-i\cdot ctg\left(\frac{\pi}{4}-i\cdot\frac{\pi}{2h}x\right) - i\cdot ctg\left(\frac{\pi}{4}+i\cdot\frac{\pi}{2h}x\right)\right] \qquad (7)$$

$$ctg(\alpha\pm\beta) = (ctg\alpha\cdot ctg\beta \mp 1)/(ctg\beta \pm ctg\alpha)$$

$$\varpi = \frac{\Gamma}{4h}\cdot\left[\frac{ctg\frac{\pi}{4}\cdot ctg\left(i\frac{\pi}{2h}x\right)+1}{ctg\left(i\frac{\pi}{2h}x\right)-ctg\frac{\pi}{4}} + \frac{ctg\frac{\pi}{4}\cdot ctg\left(i\frac{\pi}{2h}x\right)-1}{ctg\left(i\frac{\pi}{2h}x\right)+ctg\frac{\pi}{4}}\right] \qquad (8)$$

$$\varpi = \frac{\Gamma}{4h}\cdot\left[\frac{\left(ctg\left(i\cdot\frac{\pi}{2h}x\right)+1\right)^2 + \left(ctg\left(i\frac{\pi}{2h}x\right)-1\right)^2}{ctg^2\left(i\frac{\pi}{2h}x-1\right)}\right] \qquad (9)$$

setting $\alpha = i\cdot\frac{\pi}{2h}x$

$$\varpi = \frac{\Gamma}{4h} \cdot \left[\frac{ctg^2\alpha + 2 \cdot ctg\alpha + 1 + ctg^2\alpha - 2 \cdot ctg\alpha + 1}{ctg^2\alpha - 1} \right] \qquad (10)$$

$$\varpi = \frac{\Gamma}{4h} \cdot \frac{2ctg^2\alpha + 2}{ctg^2\alpha - 1} \qquad \varpi = \frac{\Gamma}{2h} \frac{1}{\sin^2\alpha \left(\frac{\cos^2\alpha}{\sin^2\alpha} - 1 \right)}$$

$$\varpi = \frac{\Gamma}{2h} \frac{1}{\cos^2\alpha - \sin^2\alpha} \qquad \varpi = \frac{\Gamma}{2h} \frac{1}{\cos^2\alpha - 1 + \cos^2\alpha}$$

$$\varpi = \frac{\Gamma}{2h} \frac{1}{2\cos^2\alpha - 1} \qquad \varpi = \frac{\Gamma}{2h} \frac{1}{\cos 2\alpha}$$

so

$$\varpi = \frac{\Gamma}{2h} \frac{1}{\cos\left(i \frac{\pi}{h} x \right)} \qquad \text{with } \cos(i\alpha) = \cosh\alpha$$

$$\varpi = \frac{\Gamma}{2h} \frac{1}{\cosh\left(\frac{\pi}{h} x \right)} = u - i \cdot v \qquad (11)$$

with v=0 at the tunnel wall

$$u = \frac{\Gamma}{2h} \cdot \frac{1}{\cosh\left(\frac{\pi}{h} x \right)} = \frac{\Gamma}{2h} \cdot \sec h\left(\frac{\pi}{h} x \right) \qquad (12)$$

x = abszissa along tunnel walls

The resulting pressure coefficient P_R for the tunnel walls is

$$P_R = \frac{(V+u)^2 - (V-u)^2}{V^2} = \frac{4u}{V}$$

where V is the free stream velocity

$$P_R = \frac{2\Gamma}{h \cdot V} \cdot \frac{1}{\cosh\left(\frac{\pi}{h} x \right)} \qquad (13)$$

The pressure distribution along the tunnel walls is integrated between the downstream position n to the upstream position m. For a point vortex on the tunnel axis with distance x from the origin the limits of integration are n – x and m – x. The lift L' associated with a point vortex is given by

$$L' = \int_{m-x}^{n-x} q_0 \cdot P_R \cdot dx \qquad (14)$$

where q_0 is the free stream dynamic pressure.

This integration of the pressure along the wall is performed by an experimental installation.

The total lift L of a point vortex is given by

$$L = \rho \cdot \Gamma \cdot V = \frac{2q_0}{V} \cdot \Gamma \qquad (15)$$

The correction factor for a point vortex at position x is then

$$\eta_x = \frac{L'}{L} = \frac{\int_{m-x}^{n-x} q_0 \cdot \frac{2\Gamma}{h \cdot V} \cdot \frac{1}{\cosh\left(\frac{\pi}{h}x\right)} dx}{\frac{2q_0 \cdot \Gamma}{V}} \qquad (16)$$

$$\eta_x = \frac{1}{h} \cdot \int_{m-x}^{n-x} \frac{1}{\cosh\left(\frac{\pi}{h}x\right)} dx \qquad \text{with} \qquad \int \frac{dx}{\cosh(x)} = 2 \cdot arctg\left(e^x\right)$$

$$\eta_x = \frac{2}{\pi} \cdot arctg\left(e^{\frac{\pi}{h}x} \right)_{m-x}^{n-x}$$

finally

$$\eta_x = \frac{2}{\pi} \cdot arctg\left[\frac{e^{\frac{-\pi x}{h}} \cdot \left(e^{\frac{\pi n}{h}} - e^{\frac{\pi m}{h}} \right)}{1 + \left(e^{\frac{-2\pi x}{h}} \cdot e^{\frac{\pi(m+n)}{h}} \right)} \right] \qquad (17)$$

In the LWK the orifices used to measure the pressure extend from m = 2,38 m to n = 2,31 m.
Tunnel height h = 2,73 m.
The correction factor for a given lift distribution is obtained from

$$\eta = \frac{\int_c P_R \cdot \eta_x \cdot d\frac{x}{c}}{\int_c P_R \cdot d\frac{x}{c}} \qquad (18) \qquad \text{with c = model chord}$$

The pressure distribution of a model can be decomposed in a basic distribution with constant pressure and correcting factor η_b and a distribution due to an angle of attack with correcting factor η_a.
According to thin-airfoil theory the pressure distribution of a flat plate is

$$P_R = \frac{4u}{V} = 4 \cdot \sin\alpha \cdot \sqrt{\frac{1-\frac{x}{c}}{\frac{x}{c}}} \quad \text{for } c_l = 1{,}0 \qquad \sin\alpha = \frac{1}{2\pi} \qquad \text{so}$$

$$P_R = \frac{2}{\pi} \cdot \sqrt{\frac{1-\frac{x}{c}}{\frac{x}{c}}} \qquad (19)$$

The correcting factor η_a can be calculated from (17) and (18).

The model is mounted with its quarter chord point at the tunnel center x=0 , y=0. In equation (17) the transformation $m' = m + c/4$ and $n' = n + c/4$ has to be performed.

The correcting factor η_b for a constant pressure $P_R = 1$ is calculated by

$$\eta_b = \frac{\int_c 1 \cdot \eta_x \cdot d\frac{x}{c}}{\int_c 1 \cdot d\frac{x}{c}} \qquad (20)$$

The lift coefficient c_l' (uncorrected for tunnel wall effects) is given in terms of the lift coefficient obtained by integration along the tunnel walls c_{lT} is given by

$$c_l' = \frac{c_{lT}}{\eta_a} - \left(\frac{\eta_a}{\eta_b} - 1\right) \cdot c_{li}$$ with the design lift coefficient c_{li}.

The correcting factors for different model chords are tabulated in table 1.

c [m]	η_a	η_b	η_b/η_a	$(\eta_b/\eta_a - 1)$
0,5	0,91413	0,91210	0,99778	-0,00222
0,7	0,91159	0,91203	1,00048	+0,00048
1,0	0,90278	0,91004	1,00804	+0,00804

Table 1

As η_a does not differ much from η_b and the influence of airfoil thickness and lift distribution is small the contribution of $\left(\frac{\eta_a}{\eta_b} - 1\right) \cdot c_{li}$ is negligible.

Thus the uncorrected lift is calculated by

$$c_l' = \frac{c_{lT}}{\eta_a} \qquad (21)$$

The agreement between lift coefficients taken from integration along tunnel walls and lift coefficients calculated from pressure orifices on the model surface confirm this method.

Experimental setup for lift measurement.

Pressure orifices along the walls of the test section opposite to the model which are positioned in a distance of 50 millimeters at half-span position and ranging from 2,39 meters upstream and 2.32 m downstream from the center of the turn tables are connected by tubes of a 1 mm diameter and a length of 145 mm to a common tube. The integrated pressures along the length L= 4,71 m are p_{Rc} along the 'ceiling' and p_{Rf} along the 'floor'. The resulting lift force l' on the tunnel walls is proportional to the difference of these pressures multiplied with the area where b is the span of the test section.

$$l' = \Delta p_{cl} \cdot L \cdot b = \left(p_{Rc} - p_{Rf}\right) \cdot L \cdot b \qquad (22)$$

The lift is

$$l' = c_l'' \cdot q_0 \cdot b \cdot c = \Delta p_{cl} \cdot L \cdot b$$

with c = model chord and q_0 is the stagnation pressure.

The lift coefficient uncorrected for the limited integration length is

$$c_l'' = \frac{\Delta p_{cl}}{q_0} \cdot \frac{L}{c} \qquad (23)$$

and

$$c_l' = \frac{c_l''}{\eta_a} = \frac{\Delta p_{cl}}{q_0} \cdot \frac{L}{c} \cdot \frac{1}{\eta_a} \qquad (24)$$

With the standard wind tunnel correction the lift coefficient is calculated by

$$c_l = \left[1 - 2\Lambda \cdot (\sigma + \xi) - \sigma\right] \frac{\Delta p_{cl}}{q_0} \cdot \frac{L}{c} \cdot \frac{1}{\eta_a} \qquad (25)$$

Measurement of drag.

The drag of an airfoil can be evaluated /2/3/4/ from the loss of total pressure in the wake. The total pressure g_w in the wake is measured by a row of small pitot tubes which form a "rake". The drag coefficient is defined by

$$c_d'' = \int_{wake} \frac{g_0 - g_w}{q_0} \cdot \frac{dy_w}{c} = \int g_c \cdot \frac{dy_w}{c} \quad (26) \quad \text{with} \quad g_c = \frac{g_0 - g_w}{q_0}$$

g_0 and q_0 are the total pressure and the stagnation pressure in front of the airfoil.

The distribution of the total pressure in the wake is simulated by a probability curve of the form

$$g_c = g_{c_{\max}} \cdot e^{-\left[\frac{B \cdot y_w}{c}\right]^2} \quad (27)$$

where $g_{c_{\max}}$ is the maximum value of g_c and B is a constant which defines the width of the wake.

$$c_d'' = \int_{wake} g_c \cdot \frac{dy_w}{c} = \int_{wake} g_{c_{\max}} \cdot e^{-Y^2} \cdot dY \quad (28)$$

where $Y = \dfrac{B \cdot y_w}{c}$ is the new variable for integration.

$$c_d'' = \int_{wake} g_c \cdot \frac{dy_w}{c} = \int_{wake} g_{c_{\max}} \cdot e^{-Y^2} \cdot dY = g_{c_{\max}} \cdot \sqrt{\pi} \quad (29)$$

If p_w is the static pressure in the wake, which is assumed to be constant, the uncorrected drag coefficient according to B.M. Jones /2 / is

$$c_d' = \int_{wake} 2\sqrt{S_w - g_c} \cdot \left(1 - \sqrt{1 - g_c} \cdot d\left(\frac{y_w}{c}\right)\right) \quad (30)$$

where $S_w = \dfrac{g_0 - p_w}{q_0}$ is the static pressure coefficient.

The correction factor K is

$$K = \frac{c_d'}{c_d''} = \frac{2}{g_{c_{\max}} \sqrt{\pi}} \int_{-\infty}^{+\infty} \sqrt{S_w - g_{c_{\max}} e^{-Y^2}} \cdot \left(1 - \sqrt{1 - g_{c_{\max}} e^{-Y^2}}\right) \cdot dY \quad (31)$$

Plots of the factor K as function of S_w and $g_{c_{\max}}$ were linearized to give

$$K\left(g_{c_{\max}}, S_w\right) = 1.018 - 0.264 \cdot g_{c\max} - 0.666 \cdot (1 - S_w) \quad (32)$$

The static pressure coefficient can be rewritten:

$$\frac{p_W - p_0}{q_0} = \frac{\Delta p_w}{q_0} = 1 - S_W \qquad (33)$$

The drag coefficient uncorrected for tunnel wall interference

$$c_d' = K \cdot \int_{wake} g_c \cdot \frac{dy_w}{c} \qquad (34)$$

can be obtained by integration of the loss of total pressure over the wake. For the calculation of K the maximum $g_{c\max}$ and the static pressure difference Δp_w must be known.
K is in the order of 0.8 to 0.9.

Experimental setup for drag measurement.

In the LWK the rake for the measurement of drag is mounted on a traversing system about 30% of chord downstream of the model. It is automatically adjusted in the middle of the wake and the flow direction. In a plane parallel to the integrating rake a row of small total head tubes is installed. These are connected to a multi-tube liquid manometer. By this means it can be observed if the wake is within the width of the integrating rake. The measurement of drag is stopped when flow separation begins.
In some cases longitudinal vortices in the boundary layers of the airfoil cause periodical variations of the drag in spanwise direction /5 /. By traversing the rake with constant velocity along the span while triggering and integrating the pressure signals a mean value of the drag is obtained.
Like the integral of the wall pressure for the measurement of lift the integral over the total pressure in the wake is gained by experimental integration. Rakes with different width are used. For model chords between 0.5 and 0.7 meters a rake with a width of 88 Millimeters is used which is built up of 56 small tubes with 0.8 mm outer and 0.6 mm inner diameter and equal lengths of 120 mm. They are soldered to a common tube with equal spaces.. In this tube the integrated pressure G_w exists. The open ends of the small tubes are thoroughly fabricated.
The static pressure p_w in the wake is measured by a small tube with static orifices and the maximum of total pressure $g_{c_{\max}}$ by a small total head tube in the middle of the rake.
Thus the correction factor K can be obtained from $\frac{\Delta p_W}{q_0} = \frac{p_w - p_0}{q_0}$ and $\Delta g_{c_{\max}} = \frac{g_0 - g_{W\max}}{q_0}$ according to equation (32). The uncorrected drag coefficient follows from

$$c_d' = K \cdot \frac{g_0 - G_w}{q_0} \cdot \frac{r}{c} \qquad (35)$$

The drag coefficient with standard tunnel corrections is

$$c_d = [1 - 2\Lambda \cdot (\sigma + \xi)] \cdot c_d' \qquad (36)$$

The experimental integration.

The lift of the model is measured by integration along the tunnel walls, also the drag is evaluated by the integration of the total pressure over the wake. These integrations are performed by an experimental procedure /4 /.
A pressure distribution P (x) along the x-axis may be given over the length Δx. By orifices at n equidistant positions which are connected to a common tube by small tubes of equal diameter d and equal length the local pressures P_i are fed to the common tube resulting in the pressure p_R. The pressure difference over each of these small tubes is P_i - p_R. The flow within the small tubes is laminar when their diameter d and length give a Reynolds number

$$\mathrm{Re} \le \mathrm{Re}_{krit} = \left(\frac{\bar{u} \cdot d}{\upsilon} \right)_{krit} = 2300 \qquad (37)$$

with $\bar{u}$ = medium velocity over the cross section of the tubes.
With these assumption the law of Hagen-Poiseulle for the flow in the tubes is

$$\bar{u} = K \cdot \Delta p = K \cdot (P_i - p_R)$$

After some time the pressures have balanced and the algebraic sum of all velocities is zero.

$$\sum_1^n \bar{u}_i = K \cdot \sum_1^n (P_i - p_R) = K \cdot \left[\sum_1^n P_i - n \cdot p_R \right] = 0 \qquad (38)$$

with n= number of the small tubes. So the resulting mean pressure is

$$p_R = \frac{1}{n} \cdot \sum_1^n P_i$$ and the integrated pressure is

$$\int_0^{\Delta x} P_i \cdot dx = p_R \cdot \Delta x \qquad (39)$$

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